Achieving sub-pixel geolocation accuracy in support of MODIS land science

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Abstract

The Moderate Resolution Imaging Spectroradiometer (MODIS) was launched in December 1999 on the polar orbiting Terra spacecraft and since February 2000 has been acquiring daily global data in 36 spectral bands—29 with 1 km, five with 500 m, and two with 250 m nadir pixel dimensions. The Terra satellite has on-board exterior orientation (position and attitude) measurement systems designed to enable geolocation of MODIS data to approximately 150 m (1\(\sigma\)) at nadir. A global network of ground control points is being used to determine biases and trends in the sensor orientation. Biases have been removed by updating models of the spacecraft and instrument orientation in the MODIS geolocation software several times since launch and have improved the MODIS geolocation to approximately 50 m (1\(\sigma\)) at nadir. This paper overviews the geolocation approach, summarizes the first year of geolocation analysis, and overviews future work. The approach allows an operational characterization of the MODIS geolocation errors and enables individual MODIS observations to be geolocated to the sub-pixel accuracies required for terrestrial global change applications.

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1. Introduction

The Moderate Resolution Imaging Spectroradiometer (MODIS) science team has developed remote sensing algorithms for deriving global time-series data products on various geophysical parameters that are used by the Earth science community (Salomonson, Barnes, Maymon, Montgomery, \& Ostrow, 1989). Accurate operational geolocation is required to generate these products; in particular, to generate the temporally composited MODIS products and to support MODIS change detection and retrieval of biophysical parameters over heterogeneous land surfaces (Justice et al., 1998; Roy, 2000; Townshend, Justice, Gurney, \& McManus, 1992). The MODIS Land Science Team requires the geolocation accuracy to be 150 m (1\(\sigma\)), with an operational goal of 50 m (1\(\sigma\)) at nadir (Nishihama et al., 1997). This accuracy requirement and goal guides the design of the MODIS geolocation algorithm and error analysis approach.

Satellite data production systems operationally register different orbits of data by geometric correction of each orbit into a common Earth-based coordinate system. Geometric correction is necessary to remove distortions introduced by the instrument sensing geometry, the curvature of the Earth, surface relief, and perturbations in the motion of the sensor relative to the surface. Geometric correction can be considered a two-stage process: first the sensed observations are geolocated, and then they are gridded into a predefined georeferenced grid. The geometric distortions present in satellite data may be categorized into system-dependent and system-independent distortions. System-dependent geometric distortions are introduced by the sensor. System-inde-
ependent distortions are introduced by the motion of the sensor, oblateness and rotation of the Earth, and surface relief. Correction for these distortions can be performed using parametric and/or nonparametric approaches. Nonparametric approaches require the identification of distinct features that have known locations, usually termed ground control points (GCPs), to model the spatial relationship between the sensed data and an Earth based coordinate system. The spatial relationships are assumed to be representative of the geometric distortions and are used to calculate mapping functions, for example polynomial functions (Bernstein, 1983). Nonparametric approaches can correct all types of geometric distortion (Roy, Devereux, Grainger, & White, 1997). However, nonparametric approaches are not suitable for the operational correction of satellite data because accurate GCPs are expensive to collect and may not be available over homogeneous, unstructured, and cloudy scenes. In addition, the sun–target–sensor geometry, used in the generation of many of the MODIS land products (e.g., Schaaf et al., 2002, this issue; Vermote, El Saleous, & Justice, 2002, this issue), must be estimated when nonparametric approaches are used (Roy & Singh, 1994). Parametric approaches require information concerning the sensing geometry (interior orientation) and the sensor attitude and position (exterior orientation), which describe the circumstances that produced the sensed image. GCPs may be used to correct errors in the sensor interior and exterior orientation knowledge (Emery & Ikeda, 1989; Moreno & Melia, 1993; Rosborough, Baldwin, & Emery, 1994). Relief information is required to remove relief distortion effects that are dependent upon the sensing altitude, the terrain height, and the distance of the terrain from nadir (Schowengerdt, 1997).

MODIS is on board the Terra, and planned Aqua, satellite which benefits from accurate and rapid measurement of the satellite exterior orientation. Consequently, the MODIS geolocation is performed using a parametric approach with GCPs only used to remove orientation biases and trends. The Terra exterior orientation is measured in real time by sensors on board the satellite. The attitude is measured by on-board inertial gyro and star-tracking sensors and the position is measured by the Tracking Data Relay Satellite System (TDRSS) On-board Navigation System (TONS) (Folta, Elrod, Lorenz, & Kapoor, 1993; Teles, Samii, & Doll, 1995). The MODIS and Terra interior orientation parameters are characterized prior to launch (Barnes, Pagano, & Salomonson, 1998; Silverman & Linder, 1992). A global digital elevation model (DEM) (Logan, 1999) is used to model and remove relief distortion effects. The MODIS geolocation product defines the geodetic latitude and longitude (WGS-84), sensor and solar geometry, slant range, and terrain height of the sensed MODIS 1 km observations. These data are subsequently used to spatially resample and temporally composite MODIS products into georeferenced grids.

Improvements to the MODIS geolocation accuracy are made by the adjustment of the sensor interior parameters with future planned work to remove systematic exterior orientation measurement errors. The information required to perform these improvements is derived by an error analysis and reduction methodology based on comparison of sensed MODIS data with a global distribution of GCPs. At the time of writing three adjustments have been made since launch. This paper overviews the geolocation approach, summarizes the first year of geolocation analysis, and overviews future work.

2. MODIS instrument geometry

The Terra, and planned Aqua, spacecraft orbit the Earth at an altitude of 705 km in a near polar orbit with an inclination of 98.2° and a mean period of 98.9 min (Salomonson et al., 1989). Terra’s sun-synchronous orbit has a dayside equatorial 10:30 am local crossing time and a 16-day repeat cycle. MODIS has a 110° across-track field of view and senses the entire equator every 2 days with full daily global coverage above approximately 30° latitude (Wolfe, Roy, & Vermote, 1998). MODIS senses in 36 spectral bands from the visible to the thermal infrared—29 with 1 km (at nadir) pixel dimensions, five with 500 m pixels, and two with 250 m pixels (Barnes et al., 1998; SBRC, 1992). MODIS is a paddle broom (sometimes called a whiskbroom) electro-optical instrument that uses the forward motion of the satellite to provide the along-track direction of scan (Fig. 1). The electromagnetic radiation (EMR) reflected or emitted from the Earth is reflected into the instrument telescope by a rotating two-sided scan mirror. One-half revolution of the scan mirror takes approximately 1.477 s and produces the across-track scanning motion. The EMR is then focused onto separate calibrated radiation detectors covered by narrow spectral band-pass filters. MODIS simultaneously senses, in each band, 10 rows of 1 km detector pixels, 20 rows of 500 m detector pixels, and 40 rows of 250 m detector pixels. Each row corresponds to a single scan line of MODIS data that is nominally composed of 1354 1 km, 2708 500 m, and 5416 250 m observations.

The MODIS detectors are grouped on four focal planes—Long Wave Infrared (LWIR), Short/Medium Wave Infrared (SWIR/MWIR), Near Infrared (NIR), and Visible (VIS) (Salomonson et al., 1989). Detectors for each band are laid out on the focal planes in the along-scan direction (Figs. 1 and 2) causing the same Earth location to be sampled at different times by different bands. The times that the MODIS scan mirror passes each of 24 encoder positions during the Earth-view portion of the scan is measured electro-optically. These timing data are subsequently used to compute the scan angle of the MODIS observations. Each 1 km, 500 m, and 250 m observation is sampled in 333.333, 166.667, and 83.333 μs, respectively. To allow for detector readout, the detector integration time is 10 μs less than the data-sampling rate at each of the three MODIS resolutions. To the first order, the MODIS point-spread function is
triangular in the scan direction (Fig. 3). The centers of the integration areas of the first observation in each scan are aligned, in a “peak-to-peak” alignment (Fig. 3). In the track direction, the point-spread function is rectangular and the observations at the different resolutions are nested, allowing four rows of 250 m observations and two rows of 500 m observations at the different resolutions.

Timing Offsets (in 1 km IFOV units) relative to the Reference Optical Axis

<table>
<thead>
<tr>
<th>Band</th>
<th>LWIR FPA</th>
<th>SWIR/MWIR FPA</th>
<th>NIR FPA</th>
<th>VIS FPA</th>
<th>Ideal Band</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>32, 31, 36, 35, 34, 33, 27, 28, 29, 30</td>
<td>23, 22, 21, 20, 5, 6, 7, 26, 24, 25</td>
<td>19, 18, 13H, 13L, 2, 1, 14H, 14L, 15, 16, 17</td>
<td>12, 11, 4, 3, 8, 9, 10</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 1. Overview of MODIS sensing geometry. A scan of MODIS data is sensed over a half revolution of the MODIS double-sided scan mirror and is focused onto four focal planes containing a total of 310, 100, and 80 detectors that sense the 1 km, 500 m, and 250 m bands, respectively (36 bands). The instantaneous sensing of the four co-registered focal planes is shown, illustrating the MODIS “paddle broom” sensing geometry.

Fig. 2. Along-scan layout of the MODIS focal planes showing the detector locations with respect to the reference optical axis. Vertically (not shown) are 10 addition sets of detectors for a total of 10, 20 and 40 detectors for each of the 1 km (bands 8–36), 500 m (bands 3–7), and 250 m (bands 1 and 2) bands, respectively, that make up a single MODIS scan. Band 0 is a hypothetical band used in the geolocation calculations.
observations to cover the same area as one row of 1 km observations.

Fig. 4 illustrates the typical coverage of three consecutive scans of the Earth’s surface. The scans are elongated because of the MODIS sensing geometry and Earth curvature such that the swath width is approximately 2340 km. The mirror angular velocity (2.127 rad/s) and the forward velocity of the satellite (7.5 km/s) are configured such that at nadir, in the track direction, the leading edge of one scan abuts the trailing edge of the next scan. Adjacent scans begin to overlap away from nadir with a 10% overlap occurring at scan angles of 24° from nadir. Consequently, the detectors on the leading edge of a scan sense surface features before the detectors on the trailing edge of the following scan. This overlap increases until at the scan edge there is an almost 50% overlap. The same point on the Earth’s surface may be sensed by up to three consecutive scans at the scan edge. This phenomenon is called the “bow-tie” effect and is seen in other whiskbroom wide-field-of-view sensors such as AVHRR (Schowengerdt, 1997), although this effect is less evident for scanners with only one detector per band. At the scan edge the projection of a MODIS detector’s instantaneous field of view (IFOV) onto the surface is approximately 2.0 and 4.8 times larger than at nadir in the track and scan directions, respectively (Wolfe et al., 1998).

3. MODIS geolocation algorithm

The MODIS geolocation calculation is performed for a hypothetical ideal band known as band 0. Band 0 is modeled as being located at the middle of the four MODIS focal planes and is used as a reference from which the positions of any band are calculated by applying appropriate offsets (Barnes et al., 1998). The nominal along-scan locations of the 36 MODIS bands and band 0 is shown in Fig. 2. The positions of the 1 km bands are registered to within 73 m (1σ) of band 0, the 500 m bands to 67 m (1σ), and the 250 m bands to 5 m (1σ). Note that because of the focal plane detector configuration, individual bands may be sensed up to 9.67 ms apart. This time difference is equivalent to the time it takes to sense 29 1 km observations (Fig. 2).

The geolocation data are computed for the centers of each Band 0 1 km observation. Computing the location of the center of a single observation is performed in several steps.
First, the line-of-sight \( \mathbf{u}_{\text{foc}} \) from each detector of a band is generated in the focal plane coordinate system:

\[
\mathbf{u}_{\text{foc}} = (x, y, f)
\]

where \((x, y)\) are the positions of the detector focal plane coordinates and \(f\) is the focal length (bold is used to denote matrices throughout this paper). The line-of-sight is then rotated from the focal plane (foc) to the telescope (tel) coordinate system and then to the instrument (inst) coordinate system:

\[
\mathbf{u}_{\text{img}} = T_{\text{inst/tel}} T_{\text{tel/foc}} \mathbf{u}_{\text{foc}}
\]

(In this paper we use the symbol \( T_{\text{sys2/sys1}} \) to represent a 3 by 3 transformation matrix that rotates a 3-vector from coordinate system sys1 to sys2. For instance, \( T_{\text{inst/tel}} \) rotates a vector from the telescope to the instrument coordinate system.) The \( T_{\text{inst/tel}} \) rotation matrix includes a rotation of the focal planes between the sensing of the detectors located at the leading and trailing edges of the focal plane (Fig. 2).

One of the key elements of the MODIS geometric model is the mirror model. Because it is impossible to manufacture a two-sided mirror with perfectly parallel sides and align the mirror perfectly with the mirror rotation axis, three angles are used to characterize the mirror surfaces and to construct the normal to the mirror surfaces (Fig. 5). The normal for each mirror side are:

\[
\hat{\mathbf{n}}_{\text{side}_i} = \begin{bmatrix}
- \sin \left( \frac{\beta}{2} + \gamma \right) \\
\sin \left( \frac{x}{2} \right) \cos \left( \frac{\beta}{2} + \gamma \right) \\
\cos \left( \frac{x}{2} \right) \cos \left( \frac{\beta}{2} + \gamma \right)
\end{bmatrix}
\]

where \(x\), \(\beta\), and \(\gamma\) are shown in Fig. 5.

Using the time of the observation, \(t\), the angle of rotation of the scan mirror \( \theta \) is determined and the normal to the rotating scan mirror \( \hat{\mathbf{n}}_{\text{side}_i} \) for mirror side \(i\) is constructed in the scan mirror (mirr) coordinate system. The mirror normal is rotated to the instrument reference frame:

\[
\hat{\mathbf{n}}_{\text{inst}} = T_{\text{inst/mirr}} T(\theta)_{\text{mirr}} \hat{\mathbf{n}}_{\text{side}_i}
\]

The intersection of the line-of-sight with the WGS-84 ellipsoid (DMA, 1987) \( \mathbf{x}_{\text{ellip}} \) is then calculated as:

\[
\mathbf{x}_{\text{ellip}} = \mathbf{p}_{\text{ellip}} + d \mathbf{u}_{\text{ellip}}
\]

The ellipsoid intersection is calculated by turning the problem into a unit sphere intersection problem (by independently rescaling the components of each vector by the inverse of the length of the corresponding ellipsoid axis) and is trigonometrically solved for the slant range \(d\).

An iterative search process is used to follow the line-of-sight from the instrument to the intersection of the terrain surface represented by a DEM. Complex relief does not
confuse this technique because the search is in a downward direction. The search begins by computing the angle \( \nu \) between the line-of-sight unit vector \( \hat{u}_{ecr} \) and ellipsoid normal \( \hat{n} \) computed at the ellipsoid intersection:

\[
\cos \nu = \hat{u}_{ecr} \cdot \hat{n}
\]

Using a precompiled maximum local terrain height, \( H_{max} \), the ECR coordinates of the search starting point \( x_{max} \) are:

\[
x_{max} = x_{ellip} + H_{max} \frac{1}{\cos \nu} \hat{u}_{ecr}
\]

Starting at \( x_{max} \), the \( i \)th iteration of the search is performed by computing the ECR coordinates of the search point:

\[
x_i = x_{max} + ds \hat{u}_{ecr}
\]

where \( ds \) is the step size. The DEM height at the latitude and longitude of each search point is calculated using bilinear interpolation. The search stops when the DEM height is higher than the search point height. The terrain intersection is then linearly interpolated from the point at which the search stopped \( x_i \) and the previous search point \( x_{i-1} \).

The height, geodetic latitude and longitude at the intersection are stored in the geolocation product. Subsequently, the slant range to the sensor, sensor zenith angle, and sensor azimuth are computed and stored for each intersection. In addition, the solar zenith and azimuth are computed from the observation time and geodetic latitude and longitude using standard astronomical models (Standish, Newhall, Williams, & Yeomans, 1992).

4. Error sources

The Terra exterior orientation is estimated from star tracker, inertial gyro and TONS navigation data streams that are combined using a Kalman filter (Folta et al., 1993). The interior orientation parameters are characterized prior to launch by the instrument and spacecraft builders (Barnes et al., 1998; Silverman & Linder, 1992). A number of error sources are anticipated that include errors in the exterior and interior orientation, digital elevation model errors, and errors due to refraction and aberration.

Errors in sensor attitude (due to exterior or interior orientation errors) will induce geolocation displacements that are directly proportional to the attitude error, the sensor altitude, local Earth curvature (ignoring terrain effects), and the scan angle (Nishihama et al., 1997). Roll attitude errors cause along-scan geolocation displacements that are asymmetric over the sensor field of view and increase from minimum displacements at nadir to maximum displacements at large scan angles. Yaw and pitch errors cause very small along-scan geolocation displacements that increase in a symmetrical nonlinear manner, from zero displacement at nadir to larger displacements for scan angles further off nadir. In the track direction, pitch errors cause displacements that are almost constant over the sensor field of view, roll errors cause no displacement, and yaw errors cause displacements that behave in a similar manner to the along-scan yaw displacements but are larger. Roll and pitch attitude errors cause the greatest geolocation displacements at nadir in the scan and track direction, respectively. For example, a 10 arc-sec error in roll causes an along-scan 34 m displacement at nadir and a 165 m error at the edge of the scan (at 55° scan angle). The same error in pitch causes a 34 m and 39 m along-track displacement at nadir and at the edge of the scan, respectively. Yaw displacements, for a 10 arc-sec error, increase from zero at nadir to 56 m in the track direction at the edge of the scan. Along and across-track sensor position measurement errors will cause directly proportional displacements in the planimetric geolocation position. Altitude position measurement errors will cause along-scan displacements that are proportional to the altitude error and increase with scan angles further from nadir.

Tables 1 and 2 summarize error estimates of the interior orientation and the exterior attitude orientation, expressed as roll, pitch, and yaw errors. These errors are broken down into dynamic and static components (Fleig, Hubanks, Storey, & Carpenter, 1993). They include errors expected

<table>
<thead>
<tr>
<th>Table 1</th>
<th>MODIS interior orientation knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic terms</td>
<td>Roll</td>
</tr>
<tr>
<td>Bearings</td>
<td>0</td>
</tr>
<tr>
<td>Mirror control system</td>
<td>12.4</td>
</tr>
<tr>
<td>Scan angle measurement</td>
<td>3.7</td>
</tr>
<tr>
<td>Neighboring EOS instrument dynamics</td>
<td>6.7</td>
</tr>
<tr>
<td>Scan mirror assembly to spacecraft interface thermal and structural distortion</td>
<td>1.7</td>
</tr>
<tr>
<td>Optical bench assembly to spacecraft interface thermal and structural distortion</td>
<td>1.7</td>
</tr>
<tr>
<td>Optical bench assembly distortion (including telescope)</td>
<td>1.7</td>
</tr>
<tr>
<td>Other dynamic errors</td>
<td>3.3</td>
</tr>
<tr>
<td>Total Dynamic (RSS)</td>
<td>15.2</td>
</tr>
<tr>
<td>Static Terms</td>
<td></td>
</tr>
<tr>
<td>Scan mirror assembly to spacecraft interface thermal and structural distortion</td>
<td>6.7</td>
</tr>
<tr>
<td>Optical bench assembly to spacecraft interface thermal and structural distortion</td>
<td>6.7</td>
</tr>
<tr>
<td>Optical bench assembly distortion (including telescope)</td>
<td>5</td>
</tr>
<tr>
<td>Spectral band registration error</td>
<td>2.4</td>
</tr>
<tr>
<td>Instrument and spacecraft mate errors</td>
<td>5</td>
</tr>
<tr>
<td>Scan mirror assembly to optical bench measurement error</td>
<td>1.7</td>
</tr>
<tr>
<td>Alignment cube angle measurement error</td>
<td>1.7</td>
</tr>
<tr>
<td>Cooler launch errors</td>
<td>2.4</td>
</tr>
<tr>
<td>Scan axis to mounting feet alignment</td>
<td>3.3</td>
</tr>
<tr>
<td>Other static errors</td>
<td>3.3</td>
</tr>
<tr>
<td>Total Static (RSS)</td>
<td>13.4</td>
</tr>
</tbody>
</table>

All errors expressed as roll, pitch and yaw errors (1σ, arc-seconds).
as a result of satellite launch and on-orbit errors due to thermal and structural distortions. It is expected that the errors summarized in Tables 1 and 2 contribute approximately 117 m (1σ) to the total geolocation error at nadir, increasing to 385 m (1σ) at 55° scan angle. In addition to these, TONS planimetric position measurement errors are expected to introduce approximately 20 m (1σ) along-track and 11 m (1σ) along-scan displacements for all scan angles. TONS position altitude measurement errors (4 m, 1σ) are expected to cause along-scan geolocation displacements from zero at nadir to 7 m (1σ) at 55° scan angle. The ratio of the dynamic and static components of these errors are estimated to be 59:41. Fig. 6 illustrates geolocation displacement error ellipses at nadir (solid lines) and at the scan edge (dotted lines) for all the error components (i.e., Tables 1 and 2, and the TONS position errors). Both the static and dynamic error components are illustrated. The along-scan component is 41% larger than the along-track component at nadir, and 249% larger at 55° scan angle. If the static errors are removed, the total remaining error is expected to be 47 m (1σ) at nadir and 166 m (1σ) at 55° scan angle.

Although the DEM is used to remove relief effects, residual geolocation errors may be introduced because of errors in the DEM, errors introduced by interpolating the line-of-sight intersection in the DEM, and high frequency relief variations occurring within each IFOV. The MODIS DEM is defined with a 30 arc-sec pixel dimension (0.925 km at the equator) and is derived from a number of different data sources including aggregated data from the National Imaging and Mapping Agency (NIMA) and Digital Chart of the World data (Gesch, Verdin, & Greenlee, 1999; Logan, 1999). The NIMA data, which covers 73% of the land surface, has a 59 m (RMSE) height error (Gesch, 1998) with other areas less accurate. DEM elevation errors will induce geolocation displacements that are proportional to the sensing altitude, the terrain height, and the distance of the terrain from nadir. For example, a 59 m DEM elevation error will cause a 129 m geolocation displacement at 55° scan angle in the scan direction, and negligible displacement (<4 m) in the track direction. Line-of-sight intersection interpolation errors, DEM resolution issues, and the variable MODIS IFOV dimensions combine to introduce complex unmodeled location errors. More study is needed to understand the nature and magnitude of these errors and the impact they may have on MODIS geolocation, particularly in areas of high spatial frequency relief.

The geolocation algorithm does not model refraction or aberration. For nominal atmospheric conditions, the refraction of visible light at 55° scan angle is equivalent to an 11 m location displacement toward nadir in the scan direction (Noerdlinger & Klein, 1995). We consider this error to be negligible compared to the increase in the along-scan-projected IFOV surface dimensions with scan angle. At nadir, light takes 2.3 ms to travel from the surface to the MODIS sensor. In this period, MODIS travels 17 m in the track direction, an effect similar to a small pitch bias of 5 arc-sec. Since we do not model refraction and aberration effects, they will appear as small biases in the geolocation error analysis.

### Table 2

<table>
<thead>
<tr>
<th>Dynamic terms</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude determination *</td>
<td>3.2</td>
<td>3.9</td>
<td>2.4</td>
</tr>
<tr>
<td>Ephemeris error *</td>
<td>0.4</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>Structure dynamics</td>
<td>1</td>
<td>2</td>
<td>1.7</td>
</tr>
<tr>
<td>Thermal distortion</td>
<td>4.3</td>
<td>4.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Total dynamic (RSS)</td>
<td>5.5</td>
<td>6.1</td>
<td>3.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Static Terms</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal distortion</td>
<td>9.1</td>
<td>13.1</td>
<td>2.9</td>
</tr>
<tr>
<td>Moisture distortion</td>
<td>5.9</td>
<td>4.6</td>
<td>1.5</td>
</tr>
<tr>
<td>Measurement error</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<td>Gravity effects</td>
<td>2.5</td>
<td>12.9</td>
<td>4.9</td>
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<tr>
<td>Launch shift</td>
<td>8</td>
<td>7.7</td>
<td>10.5</td>
</tr>
<tr>
<td>Star position knowledge *</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total static (RSS)</td>
<td>14.7</td>
<td>21.1</td>
<td>13.1</td>
</tr>
</tbody>
</table>

All errors expressed as roll, pitch and yaw errors (1σ, arc-seconds). In addition to these, planimetric position measurement errors are expected to be 20 m (1σ) along-track and 11 m (1σ) along-scan for all scan angles and with altitude position measurement errors expected to be 4 m (1σ).

![Fig. 6. Geolocation displacement error ellipses at nadir (solid lines) and at 55° scan angles (dotted lines) for two cases: when both the static and dynamic error components are considered, and when only the dynamic components errors are considered. Errors are shown at the (1σ) level and illustrate all the known expected error components (see Tables 1 and 2, and the TONS position errors described in the text).](image-url)
Fig. 7. Global distribution of 420 land ground control points (GCPs) extracted from 110 Landsat-4 and Landsat-5 TM scenes.
precision geolocated terrain corrected TM scenes has been obtained. Approximately five cloud-free GCPs are selected from each TM scene to give a total of 605 land GCPs. The latitude, longitude, and height of the GCPs are known to 15 m (1σ). A 24 km² image “chip”, including the terrain height at each 30 m TM pixel, is extracted around the GCP location and stored in a GCP library. TM bands 3 (0.66 μm) and 4 (0.83 μm), comparable to the two MODIS 250 m bands 1 (0.645 μm) and 2 (0.859 μm), are stored. The locations of the land GCPs are illustrated in Fig. 7. Approximately 50% of the TM scenes are located along US coastlines, 25% along a corridor from the Kara Sea in Russia to the southern tip of Africa, and the remainder are distributed over South America and Australia.

The Landsat GCPs are located in the sensed MODIS L1B data by area-based matching. GCPs sensed by MODIS at view zenith angles greater than 45° are not used as the surface area sensed by the MODIS IFOV increases rapidly above this zenith angle (Wolfe et al., 1998). Residual errors between the known GCP locations and the corresponding locations in the MODIS data are used in the geolocation error and reduction analyses. GCP residuals are also used to quantify the impact of changes to the interior orientation parameters and exterior orientation biases. For this latter purpose, the GCP residuals are normalized by dividing by the dimension of the local MODIS observation IFOV in the along-scan and along-track dimensions. In this way, the GCP residuals are meaningfully expressed in nadir pixel dimensions or equivalently in meters at nadir.

The GCP matching process is performed in several steps. Spatially coincident MODIS L1B data, from one of the two 250 m bands, are compared with the corresponding 24 km² TM GCP chip. The MODIS line-of-sight is initially computed for the GCP location assuming perfect MODIS geolocation. The TM chip is then spatially degraded to 250 m using a MODIS point spread function (Barnes et al., 1998) defined by the sensing geometry of each MODIS 250 m observation intersecting the chip area. An area-based correlation is computed between the MODIS 250 m and the TM degraded data using a normalized gray-level correlation technique (Pratt, 1991). This process is repeated; translating the MODIS data over a regular grid of locations centered on the line-of-sight initially computed assuming perfect MODIS geolocation. The size and resolution of the grid are defined in MODIS along-scan and across-scan angular space. The grid point where the maximum correlation occurs defines the final “true” MODIS geolocation. The difference between the true location and the initial location computed, assuming perfect MODIS geolocation, defines the geolocation residual error. In order to remove poorly matched GCPs, only those with maximum correlation coefficients greater than 0.6 are considered. In this way inaccurate, out-of-date and out-of-season GCPs, and GCPs contaminated by cloud and aerosols, are less likely to be used.

Initially, after MODIS launch, the angular equivalent of an at-nadir 2 km² grid with a 50-m sampling interval was used to perform the land GCP matching. After two updates to the MODIS interior orientation parameters, the grid size was reduced to an at-nadir equivalent of 400 m² with a 25-m sampling interval. This changed the best possible matching precision, equal to one-half the step size, from 25 to 12.5 m.

### 6. Geolocation error analysis and reduction methodology

A deterministic least squares (minimum variance) estimation is used to compute the sensor orientation parameters that best fit the GCP data. Models of the interior and exterior orientation parameters are described by linearized collinearity equations, building on the mathematical foundation described by Konecny (1976). These equations are used to compute the along-track, across-track and radial position, roll, pitch and yaw attitude angles, and mirror parameters, and their rates of change (Nishihama et al., 1997). In this process, all errors are assumed to be randomly distributed with a mean of zero. Details of the method are given in Appendix A. Initially after launch, it is not possible to uniquely differentiate between attitude and position induced geolocation errors. Consequently, the error analysis (Appendix A) is initially performed holding certain parameters fixed. Updates to the interior orientation parameters are performed by modifying look-up-tables in the geolocation software that define the transformation matrix elements used by Eqs. (1) and (2). Reliable numerical solution of the linearized collinearity equations without holding certain parameters fixed may only be performed after initial updates have been made.

Corrections of any systematic exterior orientation measurement errors can only be performed after the sensor interior orientation parameters are well defined. To date, no explicit corrections of these measurement errors have been performed, though implicit corrections may have been made when the sensor interior orientation parameters were updated. Similarly, updates of dynamic interior orientation parameters (for example, associated with day-side/night-side thermal flexing) will be performed. These updates will either necessitate temporally parameterized adjustments to the transformation matrix elements, or matrix elements

### Table 3: Interior orientation parameter updates (changes from the previous values) performed since MODIS launch

<table>
<thead>
<tr>
<th>Update data-day</th>
<th>Instrument to spacecraft alignment (arc-seconds)</th>
<th>Scan mirror coefficients (arc-seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Roll</td>
<td>Pitch</td>
</tr>
<tr>
<td>1   April 28, 2000 (2000/119)</td>
<td>243.1</td>
<td>−353.8</td>
</tr>
<tr>
<td>2   June 26, 2000 (2000/178)</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>3   March 17, 2001 (2001/077)</td>
<td>12.2</td>
<td>−6.8</td>
</tr>
</tbody>
</table>

No updates were performed to the position parameters.
parameterized with respect to the on-board instrument engineering data stream (for example, on-board temperature). Long-term changes will necessarily be removed only after sufficient time-series analysis.

7. Results

This paper was written a little over one year after first light from the MODIS instrument. At the time of writing three updates have been made to the interior orientation parameters describing the MODIS to Terra alignment and the MODIS scan mirror alignment. These updates are summarized in Table 3 and their impact on the MODIS geolocation, summarized in Table 4 and illustrated in Fig. 8. The results of these three updates are described in more detail below.

<table>
<thead>
<tr>
<th>Update</th>
<th>Number of GCP residuals</th>
<th>Along-track residuals (m)</th>
<th>Along-scan residuals (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>Measured before Update 1</td>
<td>126</td>
<td>1273</td>
<td>281</td>
</tr>
<tr>
<td>Measured after Update 1</td>
<td>258</td>
<td>116</td>
<td>383</td>
</tr>
<tr>
<td>Measured after Update 2</td>
<td>13,879</td>
<td>27</td>
<td>51</td>
</tr>
<tr>
<td>Measured after Update 3</td>
<td>414</td>
<td>18</td>
<td>38</td>
</tr>
</tbody>
</table>

These results are adjusted for scan angle and shown in nadir equivalent units.

Fig. 8. GCP residuals found immediately after launch (Table 4) and after three interior orientation updates performed in the first year of MODIS operations (Table 3). One standard deviation (1σ) error bars about the mean error are shown. These results are adjusted for scan angle and are shown in nadir equivalent units. The 150 m (1σ) geolocation accuracy specification and 50 m (1σ) operational goal are shown for comparative purposes.

In this paper geolocation errors are adjusted for scan angle and expressed in nadir equivalent units. This indicates the minimum error in meters over the MODIS field of view, i.e., as if the data were sensed at nadir. The adjustments are made by multiplying the geolocation error by the observation dimension at the scan angle and dividing by the

Fig. 9. False color images illustrating at-launch MODIS geolocation (a), and the geolocation after the first update to the MODIS interior orientation was performed (b), for a 60 km² area over the Gulf of Mexico coastline, northwest Florida. MODIS Band 1 (0.645 µm) shown as red and spatially degraded TM Band 4 (0.66 µm) shown as blue, with green set to zero. The MODIS data were sensed on February 24, 2000, and the TM data were sensed on October 3, 1993.
observation dimension at nadir. An approximation of the geolocation error at any scan angle can be calculated by multiplying the nadir equivalent error by the inverse of this observation dimension ratio. However, such estimates will only be representative of a large collection of MODIS data and not of individual MODIS observations.

7.1. Initial analyses and first update

Immediately after first Earth-view data became available, the geolocation accuracy was quantified by examination of 126 GCP residuals. This analysis gave a geolocation error estimate of 1.3 km (0.3 km, 1σ) and 1.0 km (0.4 km, 1σ) at nadir in the track and scan directions, respectively (Table 4). These geolocation errors were significantly greater than the MODIS at-launch specification and were found to be systematically mislocated. A unique solution of the linearized collinearity equations (Appendix A) based on more than a few parameters was not available. Consequently, as roll and pitch attitude errors cause the greatest geolocation displacements in the scan and track directions (at nadir), respectively, a unique solution for adjustments to just these two parameters was found by holding sensor yaw, scan mirror, and spacecraft position parameters constant (Table 3). Fig. 9 illustrates the at-launch MODIS geolocation (a), and the geolocation after this first update to the MODIS interior orientation was performed (b). The figure shows false color Landsat-5 TM data and MODIS 250 m data for a 60 km² area of the Florida coastline. The MODIS data are shown in red and the TM data are shown in blue. The TM data were spatially degraded to 250 m using the MODIS point spread function defined for the sensing geometry of each MODIS 250 m observation. The MODIS at-launch data (Fig. 9a) clearly exhibit along-track and smaller along-scan displacements relative to the TM data. Some isolated differences are also due to differences in the cloud cover and
land-cover change that may have occurred between the acquisitions of these data. Fig. 9b shows the impact of the first update. The images are well matched and differences at the 250 m resolution are difficult to discern visually.

An analysis of 258 GCP residuals, computed after the first update was performed, indicated that the update reduced the geolocation error substantially to a mean of 116 m along-track with a standard deviation of 383 m, and 19 m (146 m, 1σ) along-scan (Table 4). Fig. 10 shows along-track and along-scan GCP residuals plotted as a function of scan angle. The mirror side used to sense the L1B data containing the matched GCP location is also shown, indicating geolocation displacements that are strongly correlated with the mirror side. In the track direction (Fig. 10a), the differences between the two mirror sides are significant and the residuals are asymmetrically distributed for scan angles either side of nadir. In the scan direction (Fig. 10b), both the differences between the mirror sides and the asymmetric distributions either side of nadir are less apparent. These distributions are most likely explained by remaining yaw attitude and scan mirror coefficient errors.

7.2. Second update and time series analysis

The second update to the interior orientation parameters was performed to correct for the yaw and the mirror side differences illustrated in Fig. 10. Adjustments were made to the yaw component of the instrument to spacecraft alignment and the \( x \) and \( \beta \) scan mirror coefficients, holding the other interior orientation parameters fixed (Table 3). Fig. 11 shows the impact of the second correction, using 375 GCP residuals computed after the second update was performed. Most of the reduction in error, including the mirror side differences, is in the track direction (Fig. 11a). The geolocation accuracy after this second update was assumed to be sufficiently high to have no serious impacts on the MODIS land products in

![GCP residuals](image)

Fig. 11. GCP residuals found after the second update to the interior orientation parameters (Table 3) plotted as a function of scan angle in the track (a) and scan (b) directions. The 375 residuals are plotted to show the MODIS mirror sides used in the GCP matching process. These results are adjusted for scan angle and shown in nadir equivalent units.
production at that time. Consequently, the interior orientation parameters were not subsequently updated for a substantial period to allow a long time-series of data to be analyzed.

Figs. 12–14 show GCP residuals computed from MODIS data sensed from data-days 2000/201 (July 19, 2000) to 2001/031 (January 31, 2001). Note that data losses occurred from data-days 2000/218 to 2000/232 when no useful calibrated MODIS sensor data were available due to a problem with the MODIS on-board data formatter. Analysis of 13,879 GCPs due to a problem with the MODIS on-board data formatter. Analysis of 13,879 GCPs.

The time-series results shown in Fig. 14 exhibit a noticeable temporal trend. In the scan direction (Fig. 14b), the mean geolocation error is approximately 50 m at the beginning of the time series (data-day 2000/201), and systematically reduces to approximately 15 m by data-day 2000/362. This trend also appears in the track direction (Fig. 14a) although it is not as apparent because of high frequency residual variations. Currently, we are hypothesizing that this trend may be related to the yearly solar cycle. During this cycle, the mean...
distribution of solar illumination on the spacecraft varies as
the analemma (the ground track of the Sun at noon) changes,
and the magnitude of the incoming solar radiation varies as
the Earth–Sun distance changes. Changes in the heat distri-
bution within the spacecraft and/or sensor structure may be
influenced by these factors and may cause small pointing
knowledge errors. It is recognized that this trend may be part
of an unexplained longer-term trend and that more time-series
data are required to better understand the possible cause(s).

The along-track time-series residuals illustrated in Fig. 14
exhibit a higher temporal variation than along-scan, even
when the satellite maneuvers are not considered. It is not well
understood why these errors, primarily in the track direction,
are not observed in the scan direction. Possible explanations
for the along-track variation include a MODIS sampling
pattern that favors certain scan angles, which alias the
along-track residuals shown in Fig. 13. Analyses of the
time-series data performed by computing the mean GCP
residuals over the MODIS 16-day repeat cycle do not remove
this effect.

7.3. Third update

The third update to the interior orientation parameters was
performed, allowing all interior orientation parameters to
vary (Table 3), and assuming that position and time-varying
errors were constant. Solution of the linearized collinearity
equations (Appendix A) resulted in small instrument to
spacecraft alignment and scan mirror coefficient adjustments
(Table 3). At the time of writing, only 16 days of MODIS data
produced with the third update were available for analysis.
Using 414 ground GCPs residuals computed from these data,
the estimated along-track and along-scan geolocation error are 18 m (38 m, 1σ) and 4 m (40 m, 1σ), respectively (Table 4). Clearly, further analysis is required, using more data, before these figures can be considered reliable. In addition, the along-track and temporal variations (Figs. 13 and 14, respectively) that were not parameterized in the third update require further investigation.

8. Future work

Immediate future work will concentrate on characterizing the third interior orientation update including examination of GCP residuals over latitudinal zones to characterize within-orbit variations and autocorrelation analysis of GCP residual time-series. The geolocation software will be updated to rectify/model these effects as appropriate. In the longer term, efforts to model and remove exterior orientation measurement errors and any dynamic interior or exterior orientation errors will be made by GCP time-series analysis. The possibility of using data from multiple Terra instruments (e.g., ASTER, MISR and MODIS) to improve satellite position and attitude accuracy measurements will also be investigated.

Other important future work will include analysis and improvement of the DEM and GCP data sets. We have begun a preliminary analysis of the DEM accuracy using GCPs found in successive orbits. In the near future the Shuttle Radar Topography Mission (Hilland et al., 1998) is expected to
generate a more accurate and consistent DEM of most of the land surface. We plan to incorporate these data once they become available. We will clean the land GCP library, looking for obvious biases in GCPs or GCPs that have features that do not correlate consistently with MODIS data. Some effort will be made to incorporate additional land GCPs and refresh GCPs where necessary using Landsat-7 ETM+ data. In addition, we plan to use a global set of 6501 island GCPs originally developed for the geolocation of SeaWIFS data (Patt, Woodward, & Gregg, 1997). These island GCPs are defined as the centroid of World Vector Shoreline (WVS) island vectors with a location error of 250 m (1σ). The island GCPs will be located in the sensed MODIS L1B data by feature extraction techniques. We expect that the global island GCP library will be particularly useful in identifying and characterizing anomalous cases during maneuvers and in finding possible within-orbit variations.

MODIS data will be reprocessed several times over the mission lifetime. During each of these reprocessing activities, the most up-to-date geolocation parameters will be used to ensure that the most accurate geolocation is produced. We plan to use the same geolocation error analysis approach to support the MODIS instrument on the Aqua spacecraft, currently scheduled for launch in April 2002. The MODIS/Aqua error reduction will be performed synergistically with MODIS/Terra data and we expect the error reduction to be achieved more rapidly.

9. Conclusion

The MODIS geolocation effort has successfully met its initial objectives. We are able to provide MODIS Earth location data to sub-pixel accuracy, approaching the operational MODIS geolocation goal of 50 m (1σ) at nadir. Three updates to the parameters that define the Terra/MODIS sensing geometry have been performed in the first year since Terra launch. Provisional analysis of GCPs residuals computed after the most recent update indicate a mean geolocation error of 18 m across-track and 4 m along-scan, with standard deviations of 38 and 40 m, respectively. There are five factors that have contributed to this success. First, the Terra spacecraft was built to provide a stable platform with highly precise external orientation knowledge and very little high frequency jitter. Second, the MODIS instrument was built to provide a stable instrument with little random noise and with precise interior orientation knowledge. Third, accurate global DEM and GCP data sets were available. Fourth, GCP matching was used to determine biases in the sensor orientation. Finally, this bias information was used to provide improved processing constants for subsequent geolocation processing.

The sub-pixel accuracy provided by the MODIS geolocation product is sufficient to allow the MODIS land science team to create and analyze their science products without incurring the delay and cost associated with improving the geolocation accuracy of individual data granules. In coming years, we will look for any degradation in MODIS geolocation accuracy and attempt to eliminate seasonal variations or annual trend changes in the data to hold the geolocation accuracy within the desired range. We believe that this approach for obtaining accurate geolocation can be applied operationally to other moderate spatial resolution instruments on future missions such as the National Polar-orbiting Operational Environmental Satellite System (NPOESS) and the NPOESS Preparatory Project (NPP).

Acknowledgements

Accurate satellite geolocation requires contributions from a large number of specialized groups. We would like to recognize the contributions of the MODIS calibration and characterization team; the GSFC Flight Dynamics, Attitude Control, Terra Flight Operations and TONS groups; spacecraft builder Lockheed-Martin; instrument builder SBRS; EOS SDP toolkit engineers; and the MODIS flight operations team. The EOS DEM and GCP Science Working Groups, the EROS Data Center, the MISR and ASTER instrument teams, and the Landsat-7 project made additional contributions. We would also like to thank Bert Guindon, Hugh Kiefer, Chuck Wivell, Veljko Jovanovic, Alan Strahler, and Peter Noerdlinger for many useful suggestions in the original review of the MODIS geolocation approach. This work was performed under the direction of the MODIS Science Data Support Team and MODIS land science team in the Terrestrial Information Systems Branch (Code 922) of the Laboratory of Terrestrial Physics (Code 900) at NASA GSFC. The work was funded under NASA GSFC contracts NAS5-32350, NAS5-99085, NAS5-32373, and NASA grant NCC5449-C.

Appendix A. Numerical error analysis

This appendix describes the observation equations solved using a set of linearized collinearity equations to estimate correction biases for the satellite position, attitude, and/or scan mirror coefficients. The error analysis uses the GCP residuals expressed as the view vector shift from the observed to the true location. More details are described in Nishihama et al. (1997) and USGS (1997).

A.1. Least squares method

All estimates are performed in the orbital coordinate system. The error analysis is undertaken in the following steps: (1) define the view vector as a function of the desired parameters, (2) linearize the system and differentiate with respect to the parameters, (3) define observation equations using changes in view vectors to the ground, and (4) solve the observation equations with the least squared method.
In the least squares method, the given observations \( Y \) and the partial derivative coefficient matrix \( H \) are related to the residual errors in the parameters \( X \) by:

\[
Y = HX
\]  
(A1)

By the least squared method, we estimate the residual errors in the parameters as:

\[
X = (H^T H)^{-1} (H^T Y)
\]

The vector \( Y \) is made up of \( n \) 3-vectors \( Y^i = (dy_{1,i}, dy_{2,i}, dy_{3,i}) \), each containing the difference between the true and observed location in the orbital reference frame from GCP residual \( i \). The vector \( X \) contains the corrections \( dx_j \) to each of \( m \) parameters \( x_j \). The coefficient matrix \( H \) contains \( 3n \) rows and \( m \) columns. For each GCP residual \( i \), the 3 row by \( m \) column sub-matrix \( H' \) contains a row for each of the three components \( k \) of the observation. The row is made up of the partial derivatives \( d_y/k/dx_j \) of the component with respect to each parameter being estimated.

This method can be used to solve for all parameters simultaneously or for a subset of the parameters, holding the reminder fixed. The following sections describe derivation of the observation equations for the scan mirror coefficients, satellite position, and satellite attitude. Derivations for rates of change in these parameters are solved analytically in a similar fashion (Nishihama et al., 1997).

A.2. Scan mirror wedge angles \( \beta \) and \( \gamma \), and axis angle \( \alpha \) corrections

The normal to each mirror side are described in Eq. (1).

Let \( \delta \) be one of:

\[
\delta^1 = \frac{\beta}{2} + \gamma \quad \text{or} \quad \delta^2 = \frac{\beta}{2} - \gamma
\]  
(A2)

The normal vector in Eq. (1) containing a small change \( d\delta \) in \( \delta \) from an initial value \( \delta_0 \) can be approximated for mirror side 1 as:

\[
\mathbf{n}_1 = \mathbf{n}_1(\delta_0 + d\delta) \approx \begin{bmatrix}
- \sin \delta_0 + \cos \delta_0 \\
\sin \left( \frac{x}{2} \right) \cos \delta_0 - \sin \delta_0 \\
\cos \left( \frac{x}{2} \right) \cos \delta_0 - \sin \delta_0 \\
\end{bmatrix}
d\delta
\]

The normal vector for the mirror side 2 can be expressed similarly. The view vector in the orbital coordinate system \( \mathbf{u}_{\text{orb}} \) can be expressed as:

\[
\mathbf{u}_{\text{orb}} = \mathbf{T}_\text{orb/sc} \mathbf{T}_\text{sc/inst} \mathbf{u}_{\text{mast}} = \mathbf{T}_\text{orb/sc} \mathbf{T}_\text{sc/inst} (\mathbf{u}_{\text{img}} - 2 \mathbf{B}_n)
\]

where \( \mathbf{u}_{\text{img}} \) is the view vector in the instrument coordinate system transformed from the line-of-sight vector from a detector to a focal plane, and \( n \) is the normal vector to the mirror surface as a function of \( \delta \) (Eq. (A3)), and:

\[
\mathbf{B} = \mathbf{T}_{\text{inst/min}} \mathbf{T}(\theta)_{\text{rot}}
\]

where \( \theta \) is the mirror scan angle. Differentiating Eq. (A4) as function of \( \delta \) at the initial value \( \delta_0 \), we have:

\[
d\mathbf{u}_{\text{orb}} = Dd\delta
\]

where \( D \) is a partial derivative matrix evaluated at \( \delta_0 \).

Replacing the above notations \( d\mathbf{u}_{\text{orb}} \) and \( D \) with \( \mathbf{Y} \) and \( \mathbf{H}', \) respectively, for \( j^{\text{th}} \) GCP residual, we have:

\[
\mathbf{Y}^j = \mathbf{H}' d\delta \Rightarrow \mathbf{Y} = \mathbf{H} d\delta \Rightarrow d\delta = (\mathbf{H}'^T \mathbf{H})^{-1} (\mathbf{H}'^T \mathbf{Y})
\]

After using Eq. (A1) to separately estimate \( d\delta \) for each mirror side \( (d\delta^1, d\delta^2) \), we derive \( d\beta \) and \( d\gamma \) from relationships in Eq. (A2). Note that in Eq. (A1), \( \mathbf{H}'^T \mathbf{H} \) and \( \mathbf{H}'^T \mathbf{Y} \) are both scalar when estimating a single parameter. The derivation for \( \alpha \) follows a similar process.

A.3. Satellite position and attitude corrections

Let \( \mathbf{x}_\text{cp} = (x_{\text{cp}}, y_{\text{cp}}, z_{\text{cp}}) \) be the true location of the GCP in ECR coordinates, and \( \mathbf{p}, \mathbf{v} \) be the satellite ephemeris in ECR coordinates. Define a view vector \( \mathbf{u}_{\text{ecr}} \) from sensor to the control point by:

\[
\mathbf{u}_{\text{ecr}} = \mathbf{x}_\text{cp} - \mathbf{p}
\]

Convert all the coordinates in ECR to the orbital coordinates:

\[
\mathbf{p} \rightarrow (0, 0, 0), \quad \mathbf{x}_\text{cp} \rightarrow \mathbf{x}_0 = (x_0, y_0, z_0)
\]

\[
\mathbf{u}_{\text{ecr}} \rightarrow \mathbf{u}_{\text{orb}} = \frac{\mathbf{x}_0}{|\mathbf{x}_0|}
\]

In terms of satellite position and attitude changes, the difference between the view vector to the true GCP location \( \mathbf{u}_{\text{cp}} \) and view vector to the observed control point \( \mathbf{u}_{\text{obsrv}} \) can be expressed as sum of two parts by:

\[
\mathbf{u}_{\text{cp}} - \mathbf{u}_{\text{obsrv}} = (\mathbf{u}_{\text{cp}} - \mathbf{u}_{\text{obsrv}})_{\text{position}} + (\mathbf{u}_{\text{cp}} - \mathbf{u}_{\text{obsrv}})_{\text{attitude}}
\]

A.3.1. Satellite position

Let \( x, y, \) and \( z \) be shifts in the satellite position in the orbital coordinate system. A view vector to the GCP is given by:

\[
\mathbf{u} = \frac{(x_0 - x, y_0 - y, z_0 - z)}{\sqrt{(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2}} = (u_x, u_y, u_z)
\]
By differentiating \( u \) with respect to \( x, y \) and \( z \) changes in \( u \) can be approximated by:

\[
\begin{bmatrix}
\frac{dx}{u_{cp}} - u_{obsv} \frac{dy}{position} = -Q \\
\frac{dz}{u_{cp}} - u_{obsv} \frac{dy}{position}
\end{bmatrix}
\]

where \( Q \) is the partial derivative matrix. In Eq. (A1), for the \( i^{th} \) GCP residual, the above notations \((u_{cp} - u_{obsv})_{position}\) and matrix \( Q \) are replaced with \( Y^{i} \) and \( H^{i} \), respectively.

### A.3.2. Satellite attitude

Let \( u_{orb} \), \( u_{sc} \), and \( u_{inst} \) be a view vector in the orbital, spacecraft, and instrument coordinate systems, respectively. Then:

\[
u_{orb} = T_{orb/sc} u_{orb} = F(\xi_{r}, \xi_{p}, \xi_{y}) u_{orb}
\]

where \( \xi_{r} \), \( \xi_{p} \), and \( \xi_{y} \) are attitude parameters for roll, pitch, and yaw, respectively.

Let

\[
\begin{align*}
X_{1} &= \xi_{r} = x_{01} + x_{1}, \quad X_{2} = \xi_{p} = x_{02} + x_{2}, \\
X_{3} &= \xi_{y} = x_{03} + x_{3}
\end{align*}
\]

and

\[
T_{orb/sc} = \begin{bmatrix}
\cos X_{1} \cos X_{2} - \sin X_{1} \sin X_{2} & -\sin X_{1} \cos X_{2} - \cos X_{1} \sin X_{2} & \cos X_{1} \sin X_{2} - \sin X_{1} \cos X_{2} \\
\sin X_{1} \cos X_{2} + \cos X_{1} \sin X_{2} & \cos X_{1} \cos X_{2} - \sin X_{1} \sin X_{2} & -\cos X_{1} \sin X_{2} - \sin X_{1} \cos X_{2} \\
\sin X_{1} \sin X_{2} & \sin X_{2} \cos X_{1} & \cos X_{1} 
\end{bmatrix}
\]

Since \( x_{j} \) are small angles we can use the approximation:

\[
T_{orb/sc} \approx F_{0} + F_{1} x_{1} + F_{2} x_{2} + F_{3} x_{3}
\]

where \( F_{j} \) are \( 3 \times 3 \) partial derivate matrices with respect to each parameter. So

\[
u_{orb} = (F_{0} + F_{1} x_{1} + F_{2} x_{2} + F_{3} x_{3}) u_{orb}
\]

The observation equation can then be written as:

\[
(u_{cp} - u_{obsv})_{attitude} = \begin{bmatrix} F_{1} u_{sc} \ F_{2} u_{sc} \ F_{3} u_{sc} \end{bmatrix} \begin{bmatrix} dx_{1} \\ dx_{2} \\ dx_{3} \end{bmatrix} \]

In Eq. (A1), for \( i^{th} \) GCP residual the above notations \((u_{cp} - u_{obsv})_{attitude}\) and \([F_{1} u_{sc} F_{2} u_{sc} F_{3} u_{sc}]\) are replaced with \( Y^{i} \) and \( H^{i} \), respectively.

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